

YGB-MPI PAGE 7 - QUESTION 1

BY PYTHAGORAS ON $\triangle OMD$

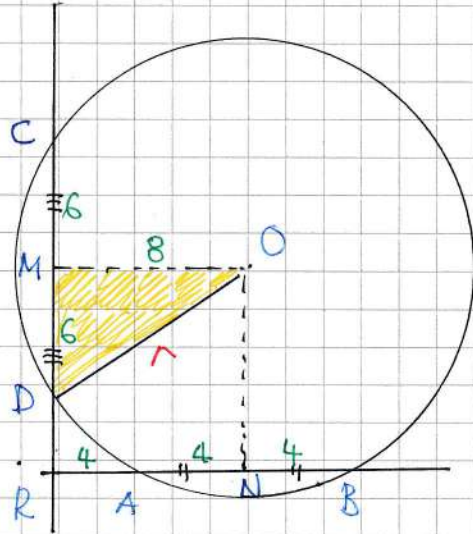
$$|OM|^2 + |MD|^2 = |OD|^2$$

$$8^2 + 6^2 = r^2$$

$$64 + 36 = r^2$$

$$r^2 = 100$$

$$r = 10$$



IVGB-MPI PAPER T-QUESTION 2

METHOD A

DEFINE A FUNCTION f & MANIPULATE IT AS FOLLOWS

$$\Rightarrow f(x, y) = x^2 - y^2 + y - x$$

$$\Rightarrow f(x, y) = (x^2 - y^2) - (x - y)$$

$$\Rightarrow f(x, y) = (x - y)(x + y) - (x - y)$$

$$\Rightarrow f(x, y) = (x - y)(x + y - 1)$$

BUT WE ARE GIVEN THAT $x + y = 1$

$$\Rightarrow f(x, y) = (x - y)(1 - 1)$$

$$\Rightarrow f(x, y) = 0$$

FOR ALL x & y SUCH THAT $x + y = 1$

$$\Rightarrow x^2 - y^2 + y - x = 0$$

$$\Rightarrow \underline{x^2 + y = y^2 + x}$$

AS REQUIRED

METHOD B

FIRSTLY IF $x = y = \frac{1}{2}$

$$x^2 + y = \left(\frac{1}{2}\right)^2 + \frac{1}{2} = \frac{1}{4} + \frac{1}{2} = \frac{3}{4}$$

$$y^2 + x = \left(\frac{1}{2}\right)^2 + \frac{1}{2} = \frac{1}{4} + \frac{1}{2} = \frac{3}{4}$$

I.E THE RESULT HOLDS IF $x = y = \frac{1}{2}$

1YGB - MPI PART 2 T-QUESTION 2

NEXT SUPPOSE $x \neq y$ so $x - y \neq 0$

$$\Rightarrow x + y = 1$$

$$\Rightarrow (x - y)(x + y) = 1(x - y)$$

$$\Rightarrow x^2 - y^2 = x - y$$

$$\Rightarrow x^2 + y^2 = y^2 + x$$

AS REQUIRED FOR ALL x & y

- 1 -

YGB - MATH PAPER T - QUESTION 3

● USING THE "DIFFERENCE OF CUBES" IDENTITY

$$a^3 - b^3 \equiv (a - b)(a^2 + ab + 1)$$

$$(\sqrt[3]{4})^3 - 1^3 = (\sqrt[3]{4} - 1) [(\sqrt[3]{4})^2 + \sqrt[3]{4} + 1]$$

$$4 - 1 = (\sqrt[3]{4} - 1) [\sqrt[3]{16} + \sqrt[3]{4} + 1]$$

● HENCE WE CAN MANIPULATE THE EXPRESSION AS FOLLOWS

$$\frac{3}{\sqrt[3]{4} - 1} = \frac{3(\sqrt[3]{16} + \sqrt[3]{4} + 1)}{(\sqrt[3]{4} - 1)(\sqrt[3]{16} + \sqrt[3]{4} + 1)}$$

$$= \frac{3(\sqrt[3]{16} + \sqrt[3]{4} + 1)}{\cancel{3}}$$

$$= \sqrt[3]{16} + \sqrt[3]{4} + 1$$

//

-1-

1YGB - MPI PAPER T - QUESTION 4

$$f(x) = \frac{1}{6}x^2 + 3x + 12$$

FACTORIZING BY COMPLETING THE SQUARE (OR TREAT IT AS THE EQUATION $f(x)=0$ & USE THE QUADRATIC FORMULA)

$$\Rightarrow f(x) = \frac{1}{6} [x^2 + 18x + 72]$$

$$\Rightarrow f(x) = \frac{1}{6} [(x+9)^2 - 81 + 72]$$

$$\Rightarrow f(x) = \frac{1}{6} [(x+9)^2 - 9]$$

$$\Rightarrow f(x) = \frac{1}{6} [(x+9)^2 - 3^2]$$

$$\Rightarrow f(x) = \frac{1}{6} [(x+9+3)(x+9-3)]$$

$$\Rightarrow f(x) = \frac{1}{6} (x+12)(x+6)$$

(NATURALLY WE COULD HAVE FACTORIZED DIRECTLY TO THIS IF WE SPOTTED IT.)

NOW THIS CAN BE WRITTEN 4 DIFFERENT WAYS AS A PRODUCT OF TWO LINEAR FACTORS

- $f(x) = (\frac{1}{6}x + 2)(x+6)$

- $f(x) = (x+12)(\frac{1}{6}x+1)$

- $f(x) = (\frac{1}{2}x+6)(\frac{1}{3}x+2)$

- $f(x) = (\frac{1}{3}x+4)(\frac{1}{2}x+3)$

1YGB - MPI PAPER T - QUESTION 5

- START BY ATTEMPTING TO CREATE AN EQUATION IN A SINGLE TRIGONOMETRIC FUNCTION, AS FOLLOWS

$$\Rightarrow 4\cos^2\theta + \tan^4\theta = 10$$

$$\Rightarrow 4\cos^2\theta + \frac{\sin^4\theta}{\cos^4\theta} = 10$$

$$\Rightarrow 4\cos^6\theta + \sin^4\theta = 10\cos^4\theta$$

$$\Rightarrow 4\cos^6\theta + (1 - \cos^2\theta)^2 = 10\cos^4\theta$$

$$\Rightarrow 4\cos^6\theta + 1 - 2\cos^2\theta + \cos^4\theta = 10\cos^4\theta$$

$$\Rightarrow 4\cos^6\theta - 9\cos^4\theta - 2\cos^2\theta + 1 = 0$$

THIS IS A CUBIC IN $\cos^2\theta$ — LET $a = \cos^2\theta$

$$\Rightarrow 4a^3 - 9a^2 - 2a + 1 = 0$$

- WE ARE GIVEN THAT $\theta = \frac{\pi}{3}$ IS A SOLUTION

$$\Rightarrow \cos \frac{\pi}{3} = \frac{1}{2}$$

$$\Rightarrow \cos^2 \frac{\pi}{3} = \frac{1}{4}$$

$$\Rightarrow a = \frac{1}{4}$$

$$\Rightarrow 4a - 1 = 0$$

$\therefore (4a - 1)$ IS A FACTOR OF THE CUBIC

- BY LONG DIVISION OR MANIPULATIONS

$$\Rightarrow a^2(4a - 1) - 2a(4a - 1) - (4a - 1) = 0$$

$$\Rightarrow (4a - 1)(a^2 - 2a - 1) = 0$$

1YGB - MPI PAPER T- QUESTION 5

● NOW LOOKING AT THE QUADRATIC

$$\Rightarrow a^2 - 2a - 1 = 0$$

$$\Rightarrow (a-1)^2 - 2 = 0$$

$$\Rightarrow (a-1)^2 = 2$$

$$\Rightarrow a-1 = \pm\sqrt{2}$$

$$\Rightarrow a = 1 \pm \sqrt{2}$$

$$\Rightarrow \cos^2 \theta = \begin{cases} 1 + \sqrt{2} > 1 \\ 1 - \sqrt{2} < 0 \end{cases}$$

$$\left[\cos \theta = \begin{cases} \sqrt{1 + \sqrt{2}} > 1 \\ -\sqrt{1 + \sqrt{2}} < -1 \end{cases} \right]$$

● Thus only solution is $a = \frac{1}{4}$

$$\Rightarrow \cos^2 \theta = \frac{1}{4}$$

$$\Rightarrow \cos \theta = \begin{cases} \frac{1}{2} \\ -\frac{1}{2} \end{cases}$$

● Thus $\cos \theta = \frac{1}{2}$

$$\theta = \frac{\pi}{3}, \frac{5\pi}{3}$$

$$\cos \theta = -\frac{1}{2}$$

$$\theta = \frac{2\pi}{3}, \frac{4\pi}{3}$$

IVGB - MPI PAPER T - QUESTION 6

STARTING WITH A DIAGRAM

$$\triangle ABE \sim \triangle BCF$$

$$\frac{y}{6} = \frac{4}{x}$$

$$xy = 24$$

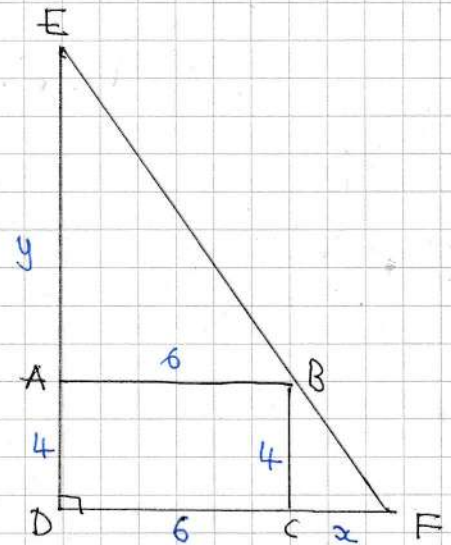
GET AN EXPRESSION FOR THE AREA OF THE TRIANGLE DEF

$$A = \frac{1}{2}(x+6)(y+4)$$

$$A = \frac{1}{2}(x+6)\left(\frac{24}{x}+4\right)$$

$$A = \frac{1}{2}\left(24 + 4x + \frac{144}{x} + 24\right)$$

$$A = 24 + 2x + \frac{72}{x}$$



DIFFERENTIATE "A" w.r.t x & SOLVE FOR ZERO

$$\frac{dA}{dx} = 2 - \frac{72}{x^2}$$

$$0 = 2 - \frac{72}{x^2}$$

$$2x^2 = 72$$

$$x^2 = 36$$

$$x = +6$$

$$\therefore A \Big|_{x=6} = 24 + 2 \times 6 + \frac{72}{6} = 24 + 12 + 12 = 48$$

JUSTIFYING IT IS MINIMUM

$$\frac{d^2A}{dx^2} = \frac{216}{x^3}$$

$$\frac{d^2A}{dx^2} \Big|_{x=6} = \frac{216}{6^3} > 0 \quad \text{INDEED MINIMUM AREA}$$

IYGB - MPI PAPER T - QUESTION 7

EXPAND IN GENERAL FORM UP TO x^2

$$\begin{aligned}(a+bx)^n &= \binom{n}{0} a^n (bx)^0 + \binom{n}{1} a^{n-1} (bx)^1 + \binom{n}{2} a^{n-2} (bx)^2 + \dots \\&= [1 \times a^n \times 1] + [n \times a^{n-1} \times bx] + \left[\frac{n(n-1)}{1 \times 2} \times a^{n-2} \times b^2 x^2 \right] + \dots \\&= a^n + [na^{n-1}b]x + \left[\frac{1}{2}n(n-1)a^{n-2}b^2 \right]x^2\end{aligned}$$

\uparrow 8192 \uparrow 6656 \uparrow 2496

COLLECTING THE EQUATIONS

(I) - $a^n = 8192$

(II) - $na^{n-1}b = 6656 \quad \leftarrow \text{SQUARE}$

(III) - $\frac{1}{2}n(n-1)a^{n-2}b^2 = 2496 \quad \leftarrow \text{DOUBLE}$

$$\Rightarrow \begin{cases} n^2 a^{2n-2} b^2 = 6656^2 \\ n(n-1) a^{n-2} b^2 = 4992 \end{cases}$$

$$\Rightarrow \frac{n^2 \cancel{a^{2n}} \times \cancel{a^{-2}} \times \cancel{b^2}}{n(n-1) \times \cancel{a^n} \times \cancel{a^{-2}} \times \cancel{b^2}} = \frac{6656^2}{4992}$$

$$\Rightarrow \frac{n}{n-1} \times a^n = \frac{26624}{3}$$

$$\Rightarrow \frac{n}{n-1} \times 8192 = \frac{26624}{3}$$

\downarrow (I)

$$\Rightarrow \frac{n}{n-1} = \frac{13}{12}$$

1YGB - MPI PAPER T - QUESTION 7

$$\Rightarrow 12n = 13n - 13$$

$$\Rightarrow 13 = n$$

$$\text{It } n=13 //$$

NOW USING $a^n = 8192$ (I)

$$\Rightarrow a^{13} = 8192$$

$$\Rightarrow a = \sqrt[13]{8192}$$

$$\Rightarrow a = 2 //$$

FINALLY USING $na^{n-1}b = 6656$

$$\Rightarrow 13 \times 2^{12} \times b = 6656$$

$$\Rightarrow b = \frac{6656}{13 \times 2^{12}}$$

$$\Rightarrow b = \frac{1}{8} //$$

YGB-MPI PAPER T - QUESTION 8

USING CALCULUS, NOTING k IS A CONSTANT

$$f(x) = x^2 + 6x + 20 + k(x^2 - 3x - 12)$$

$$f'(x) = 2x + 6 + k(2x - 3)$$

Now $f'(-2) = 0$, so we have

$$0 = -4 + 6 + k(-4 - 3)$$

$$-2 = -7k$$

$$k = \frac{2}{7}$$

Finally $f(-2) = p$, so we have

$$p = 4 - 12 + 20 - \frac{2}{7}(4 + 6 - 12)$$

$$p = 12 + \frac{2}{7}(-2)$$

$$p = \frac{80}{7}$$

ALTERNATIVE BY COMPLETING THE SQUARE AS $f(x)$ IS A QUADRATIC

$$f(x) = (k+1)x^2 + (6-3k)x + (20-12k)$$

$$f(x) = (k+1) \left[x^2 + \frac{6-3k}{k+1}x + \frac{20-12k}{k+1} \right]$$

$$f(x) = (k+1) \left[\left[x + \frac{6-3k}{2(k+1)} \right]^2 - g(k) \right]$$

NOW WE HAVE

$$\frac{6-3k}{2(k+1)} = 2$$

1YGB MPI PART 1 - QUESTION 8

$$\Rightarrow 6 - 3k = 2 \times 2(k+1)$$

$$\Rightarrow 6 - 3k = 4k + 4$$

$$\Rightarrow 2 = 7k$$

$$\Rightarrow k = \frac{2}{7}$$

AND THE VALUE OF P SHOWS

LYGB - MPI PAPER T - QUESTION 9

START WITH A DIAGRAM — THEN OBTAIN SOME STANDARD INFO

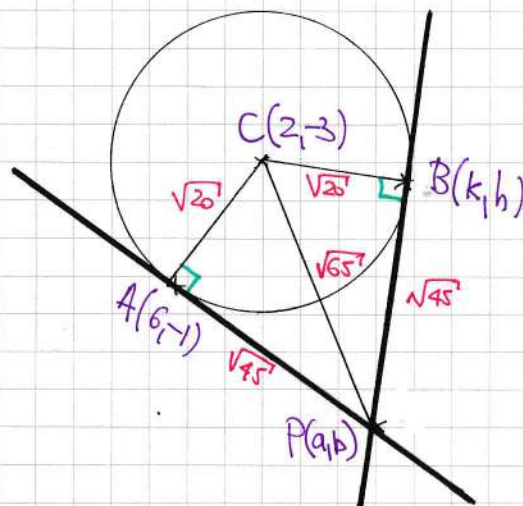
$$\Rightarrow x^2 + y^2 - 4x + 6y = 7$$

$$\Rightarrow x^2 - 4x + y^2 + 6y = 7$$

$$\Rightarrow (x-2)^2 - 4 + (y+3)^2 - 9 = 7$$

$$\Rightarrow (x-2)^2 + (y+3)^2 = 20$$

$$\underline{C(2, -3)}, \quad \underline{r = \sqrt{20}}$$



TRIVIAL BY PYTHAGORAS AND CIRCLE
GEOMETRY $|AP| = |BP| = \sqrt{45}$

LET $P(a, b)$ & $B(k, h)$, AND MARK ALL KNOWN INFORMATION
IN THE DIAGRAM

$$\Rightarrow \text{"GRAD } AC" \times \text{"GRAD } AP" = -1$$

$$\Rightarrow \frac{-1 - (-3)}{6 - 2} \times \frac{b - (-1)}{a - 6} = -1$$

$$\Rightarrow \frac{2}{4} \times \frac{b+1}{a-6} = -1$$

$$\Rightarrow \frac{b+1}{a-6} = -2$$

$$\Rightarrow b+1 = -2a+12$$

$$\Rightarrow \underline{b = 11 - 2a}$$

NOW WE HAVE DISTANCE CONSTRAINTS, I.E. $|PA| = \sqrt{45}$ & $|PC| = \sqrt{65}$

$$\left. \begin{array}{l} (a-6)^2 + (b+1)^2 = 45 \\ (a-2)^2 + (b+3)^2 = 65 \end{array} \right\} \Rightarrow \text{USING EITHER EQUATION} \\ \text{WITH } b = 11 - 2a$$

IYGB-MPI PART T-QUESTION 9

$$\Rightarrow (a-6)^2 + [(11-2a)+1]^2 = 45$$

$$\Rightarrow a^2 - 12a + 36 + (12-2a)^2 = 45$$

$$\Rightarrow a^2 - 12a - 9 + (144 - 48a + 4a^2) = 0$$

$$\Rightarrow 5a^2 - 60a + 135 = 0$$

$$\Rightarrow a^2 - 12a + 27 = 0$$

$$\Rightarrow (a-9)(a-3) = 0$$

$$\Rightarrow a = \begin{matrix} 3 \\ 9 \end{matrix} \quad b = \begin{matrix} 5 \\ -7 \end{matrix}$$

$$\therefore \underline{P(3,5) \text{ OR } P(9,-7)}$$

NOW LOOKING AT THE TRIANGLE $\triangle BCP$

USE $P(3,5)$ FIRST

$$\underline{|BP| = \sqrt{45} \quad \& \quad |BC| = \sqrt{20}}$$

$$\Rightarrow \left. \begin{aligned} (k-3)^2 + (h-5)^2 &= 45 \\ (k-2)^2 + (h+3)^2 &= 20 \end{aligned} \right\}$$

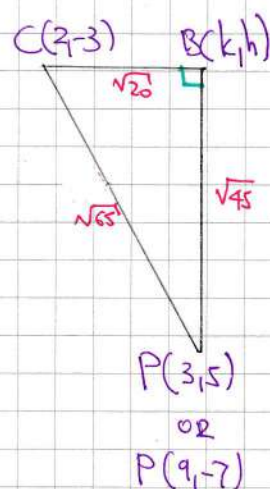
$$\Rightarrow k^2 - 6k + 9 + h^2 - 10h + 25 = 45$$

$$k^2 - 4k + 4 + h^2 + 6h + 9 = 20$$

$$\underline{-2k + 5 \quad -16h + 16 = 25}$$

SUBTRACT

$$\Rightarrow -2k - 16h - 4 = 0$$



IYGB-MPI PAPER T-QUESTION 9

$$\Rightarrow -k - 8h - 2 = 0$$

$$\Rightarrow k = -2 - 8h$$

$$\text{SUBSTITUTE INTO } (k-2)^2 + (h-3)^2 = 20$$

$$\Rightarrow [(-2-8h)-2]^2 + (h+3)^2 = 20$$

$$\Rightarrow [-4-8h]^2 + (h+3)^2 = 20$$

$$\Rightarrow (8h+4)^2 + (h+3)^2 = 20$$

$$\Rightarrow 64h^2 + 64h + 16 + h^2 + 6h + 9 = 20$$

$$\Rightarrow 65h^2 + 70h + 5 = 0$$

$$\Rightarrow 13h + 14h + 1 = 0$$

$$\Rightarrow (13h+1)(h+1) = 0$$

$$h = \begin{cases} -1 \\ -\frac{1}{13} \end{cases}$$

$$k = \begin{cases} 6 \\ -\frac{18}{13} \end{cases}$$

$$\therefore \underline{B(-\frac{1}{13}, -\frac{18}{13})} \quad \& \quad A(-1, 6) \text{ AS EXPECTED}$$

USE P(9, -7) NEXT

USING $|BP| = \sqrt{45}$ & $|BC| = \sqrt{20}$ AND SMT DIAGRAM

$$\Rightarrow \left. \begin{aligned} (k-9)^2 + (h+7)^2 &= 45 \\ (k-2)^2 + (h+3)^2 &= 20 \end{aligned} \right\}$$

$$\Rightarrow \begin{aligned} k^2 - 18k + 81 + h^2 + 14h + 49 &= 45 \\ k^2 - 4k + 4 + h^2 + 6h + 9 &= 20 \\ \hline -14k + 77 + 8h + 40 &= 25 \end{aligned}$$

SUBTRACT

1YGB - MPI PAPER T - QUESTION 9

$$\Rightarrow -14k + 8h + 92 = 0$$

$$\Rightarrow -7k + 4h + 46 = 0$$

$$\Rightarrow \underline{4h = 7k - 46}$$

PROCEED BY THE SUBSTITUTION INTO $(k-2)^2 + (h+3)^2 = 20$

$$\Rightarrow (k-2)^2 + (h+3)^2 = 20$$

$$\Rightarrow 16(k-2)^2 + 16(h+3)^2 = 320$$

$$\Rightarrow 16(k^2 - 4k + 4) + (4h + 12)^2 = 320$$

$$\Rightarrow 16k^2 - 64k + 64 + (7k - 46 + 12)^2 = 320$$

$$\Rightarrow 16k^2 - 64k + 64 + (7k - 34)^2 = 320$$

$$\Rightarrow 16k^2 - 64k + 64 + 49k^2 - 476k + 1156 = 320$$

$$\Rightarrow 65k^2 - 540k + 900 = 0$$

$$\Rightarrow 13k^2 - 108k + 180 = 0$$

$$\Rightarrow (13k - 30)(k - 6) = 0$$

\xrightarrow{R} FROM POINT A

$$\Rightarrow k = \begin{cases} 6 \\ \frac{30}{13} \end{cases}$$

$$h = \begin{cases} -1 \\ -\frac{97}{13} \end{cases}$$

\leftarrow POINT A

\leftarrow POINT B

THENCE THE REQUIRED ANSWERS ARE

EITHER $P(3, 5)$ $B(-\frac{1}{13}, -\frac{10}{13})$

OR $P(9, 7)$ $B(\frac{30}{13}, -\frac{97}{13})$

1YGB - MPI PAPER T - QUESTION 10

$$(1 + e^{-2})(e^2)^{x^2-4x+5} = e^2 + e^{4(x-2)^2}$$

TIDY THE EQUATION AS FOLLOWS

$$\Rightarrow (1 + e^{-2}) e^{2(x^2-4x+5)} = e^2 + e^{4(x-2)^2}$$

$$\Rightarrow (1 + e^{-2}) e^{2[(x-2)^2+1]} = e^2 + e^{4(x-2)^2}$$

$$\Rightarrow (1 + e^{-2}) e^{2(x-2)^2+2} = e^2 + e^{2 \times 2(x-2)^2}$$

$$\Rightarrow (1 + e^{-2}) e^{2(x-2)^2} \times e^2 = e^2 + [e^{2(x-2)^2}]^2$$

$$\Rightarrow (e^2 + 1) e^{2(x-2)^2} = e^2 + [e^{2(x-2)^2}]^2$$

$$\Rightarrow (e^2 + 1) A = e^2 + A^2$$

$$A = e^{2(x-2)^2}$$

REGROUP & FACTORIZE THE QUADRATIC

$$\Rightarrow 0 = A^2 - (e^2 + 1)A + e^2$$

$$\Rightarrow 0 = (A - e^2)(A - 1)$$

$$\Rightarrow A = \begin{matrix} 1 \\ < \\ e^2 \end{matrix}$$

$$\Rightarrow e^{2(x-2)^2} = \begin{matrix} 1 \\ < \\ e^2 \end{matrix}$$

$$\Rightarrow \text{EITHER } x=2$$

$$\text{OR } 2(x-2)^2 = 2$$

$$(x-2)^2 = 1$$

$$x-2 = \begin{matrix} -1 \\ < \\ +1 \end{matrix}$$

$$x = \begin{matrix} 1 \\ < \\ 3 \end{matrix}$$

1YGB - MPI PAPER T - QUESTION 11

STARTING WITH A DIAGRAM - NOT TO ANY SCALE

OBTAIN THE CO-ORDINATES OF R

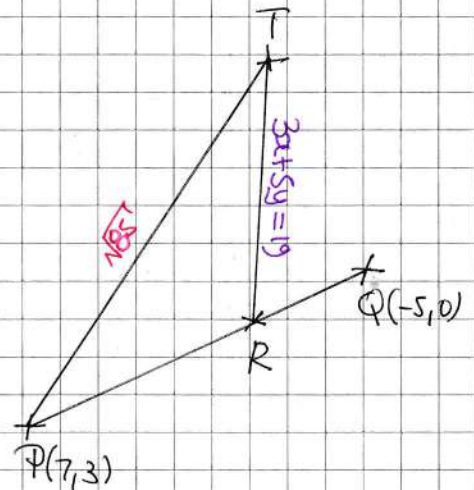
• GRADIENT PQ = $\frac{0-3}{-5-7} = \frac{-3}{-12} = \frac{1}{4}$

• EQUATION PQ: $y - y_0 = m(x - x_0)$
 $y - 0 = \frac{1}{4}(x + 5)$
 $y = \frac{1}{4}x + \frac{5}{4}$
 $4y = x + 5$

• SOLVING SIMULTANEOUSLY

$$\left. \begin{array}{l} 4y = x + 5 \\ 3x + 5y = 19 \end{array} \right\} \Rightarrow x = 4y - 5$$
$$\Rightarrow 3(4y - 5) + 5y = 19$$
$$\Rightarrow 17y - 15 = 19$$
$$\Rightarrow 17y = 34$$
$$\Rightarrow y = 2 \quad \& \quad x = 3$$

$\therefore R(3, 2)$



NOW PROCEED AS FOLLOWS

• LET $T(a, b)$

• $|PT| = \sqrt{45} \Rightarrow \sqrt{(a-7)^2 + (b-3)^2} = \sqrt{45}$
 $\Rightarrow (a-7)^2 + (b-3)^2 = 45$

• BUT AS T LIES ON $3x + 5y = 19 \Rightarrow 3a + 5b = 19$

COMBINING RESULTS

$$\left. \begin{array}{l} 3a + 5b = 19 \\ (a-7)^2 + (b-3)^2 = 45 \end{array} \right\} \Rightarrow 3a = 19 - 5b$$
$$\left. \begin{array}{l} 3a = 19 - 5b \\ 9(a-7)^2 + 9(b-3)^2 = 45 \times 9 \end{array} \right\} \Rightarrow (3a-21)^2 + 9(b-3)^2 = 765$$

$$\Rightarrow (19 - 5b - 21)^2 + 9(b-3)^2 = 765$$

$$\Rightarrow (-5b - 2)^2 + 9(b-3)^2 = 765$$

1YGB - MPI PART 1 - QUESTION 11

$$\Rightarrow (5b+2)^2 + 9(b-3)^2 = 765$$

$$\Rightarrow 25b^2 + 20b + 4 + 9b^2 - 54b + 81 = 765$$

$$\Rightarrow 34b^2 - 34b - 680 = 0$$

$$\Rightarrow b^2 - b - 20 = 0$$

$$\Rightarrow (b+4)(b-5) = 0$$

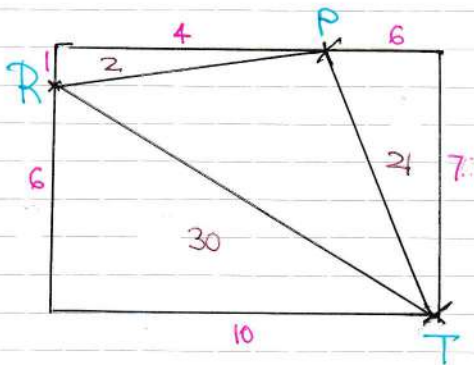
$$a = \frac{19-5b}{3}$$

$$\Rightarrow b = \begin{matrix} -4 \\ 5 \end{matrix} \quad a = \begin{matrix} 13 \\ -2 \end{matrix}$$

$$\underline{T(13, -4)} \quad \text{OR} \quad \underline{T(-2, 5)}$$

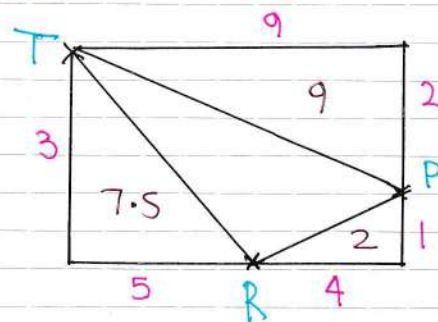
BOTH POSITIONS ARE POSSIBLE

P(7,3) R(3,2) T(13,-4)



$$\begin{aligned} \text{AREA OF } \triangle PRT &= (7 \times 10) - 30 - 21 - 2 \\ &= 17 \end{aligned}$$

P(7,3) R(3,2) T(-2,5)



$$\begin{aligned} \text{AREA OF } \triangle PRT &= (9 \times 3) - 9 - 2 - 7.5 \\ &= 27 - 11 - 7.5 \\ &= 16 - 7.5 \\ &= 8.5 \end{aligned}$$

INDDED TWO POSSIBLE AREAS OF WHICH ONE IS TWICE AS LARGE AS THE OTHER

NOTE THAT THE AREA OF A TRIANGLE WITH VERTICES AT (x_1, y_1) , (x_2, y_2) , (x_3, y_3) CAN ALSO BE FOUND BY THE FORMULA

$$\text{AREA} = \frac{1}{2} \begin{vmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \end{vmatrix}$$

HYGB - MPI PAPER T - QUESTION 12

SOLVING SIMULTANEOUSLY

$$\left. \begin{array}{l} y = \frac{3}{4}x^2 - 4\sqrt{x} + 7 \\ y = 5x - 9 \end{array} \right\} \Rightarrow \frac{3}{4}x^2 - 4x^{\frac{1}{2}} + 7 = 5x - 9$$
$$\Rightarrow 3x^2 - 16x^{\frac{1}{2}} + 28 = 20x - 36$$
$$\Rightarrow 3x^2 - 20x - 16x^{\frac{1}{2}} + 64 = 0$$

USING A SUBSTITUTION, $t = x^{\frac{1}{2}}$

$$\Rightarrow 3t^4 - 20t^2 - 16t + 64 = 0$$

$$\Rightarrow f(t) = 3t^4 - 20t^2 - 16t + 64$$

$$\Rightarrow f'(t) = 12t^3 - 40t - 16 = 4[3t^3 - 10t - 4]$$

AS WE ARE LOOKING FOR A POINT OF TANGENCY WE ARE LOOKING FOR A SOLUTION FOR BOTH

$$\underline{3t^4 - 20t^2 - 16t + 64 = 0} \quad \& \quad \underline{3t^3 - 10t - 4 = 0}$$

• $t=1$	$3 - 20 - 16 + 64 \neq 0$	$3 - 10 - 4 \neq 0$
• $t=-1$	$3 - 20 + 16 + 64 \neq 0$	$-3 + 10 - 4 \neq 0$
• $t=2$	$48 - 80 - 32 + 64 = 0$	$24 - 20 - 4 = 0$

∴ $t=2$ YIELDS A REPEATED ROOT

BY LONG DIVISION OR EQUIVALENT METHOD

$$\begin{array}{r} 3t^2 + 12t + 16 \\ t^2 - 4t + 4 \overline{) 3t^4 + 0t^3 - 20t^2 - 16t + 64} \\ \underline{-3t^4 + 12t^3 - 12t^2} \\ 12t^3 - 32t^2 - 16t + 64 \\ \underline{-12t^3 + 48t^2 - 48t} \\ 16t^2 - 64t + 64 \\ \underline{-16t^2 + 64t - 64} \\ 0 \end{array}$$

$(t-2)^2$

1Y0B - MPI PAPER T - QUESTION 12

COLLECTING THE RESULTS

$$\Rightarrow 3t^4 - 20t^2 - 16t + 64 = 0$$

$$\Rightarrow (t-2)^2(3t^2 + 12t + 16) = 0$$



$$\begin{aligned} b^2 - 4ac &= 12^2 - 4 \times 3 \times 16 \\ &= 144 - 192 \\ &= -48 < 0 \end{aligned}$$

∴ ONLY SOLUTION IS THE POINT OF TANGENCY

$$t = 2 \Rightarrow \sqrt{x} = 2$$

$$\Rightarrow x = 4$$

$$\Rightarrow y = 11$$

∴ $y = 5x - 9$ IS A TANGENT AT $(4, 11)$ & DOES NOT MEET THE CURVE AGAIN

ALTERNATIVE METHOD

START BY DIFFERENTIATION, TO FIND WHERE $\frac{dy}{dx} = 5$

$$\Rightarrow y = \frac{3}{4}x^2 - 4x^{\frac{1}{2}} + 7$$

$$\Rightarrow \frac{dy}{dx} = \frac{3}{2}x - 2x^{-\frac{1}{2}}$$

$$\Rightarrow 5 = \frac{3}{2}x - \frac{2}{\sqrt{x}}$$

$$\Rightarrow 10 = 3x - \frac{4}{\sqrt{x}}$$

$$\Rightarrow 10x^{\frac{1}{2}} = 3x^{\frac{3}{2}} - 4$$

× 2

× $\sqrt{x} = x^{\frac{1}{2}}$

GRADIENT OF
THE GIVEN LINE

YGB-MPI PAPER T - QUESTION 12

$$\Rightarrow 3x^{\frac{3}{2}} - 10x^{\frac{1}{2}} - 4 = 0$$

$$\Rightarrow 3t^3 - 10t - 4 = 0 \quad (t = x^{\frac{1}{2}})$$

$$\Rightarrow f(t) \equiv 3t^3 - 10t - 4$$

LOOK FOR SOLUTIONS BY INSPECTION

$$f(1) = 3 - 10 - 4 \neq 0$$

$$f(-1) = -3 + 10 - 4 \neq 0$$

$$f(2) = 24 - 20 - 4 = 0$$

$$\Rightarrow (t-2)(\underbrace{3t^2}_{-6t^2} + \underbrace{At}_{At^2} + 2) = 0$$

$$\begin{aligned} \because At^2 - 6t^2 &= 0 \\ t^2(A-6) &= 0 \\ A &= 6 \end{aligned}$$

$$\Rightarrow (t-2)(3t^2 - 6t + 2) = 0$$

$$b^2 - 4ac = 36 - 4 \times 3 \times 2 = 12 > 0$$

$$t = \frac{6 \pm \sqrt{12}}{6} = 1 \pm \frac{1}{3}\sqrt{3}$$

$$t = 1 \pm \frac{1}{3}\sqrt{3}$$

$$\Rightarrow \underline{t} = x^{\frac{1}{2}} = \begin{cases} 2 \\ 1 + \frac{1}{3}\sqrt{3} \\ 1 - \frac{1}{3}\sqrt{3} \end{cases}$$

$$\Rightarrow \underline{x} = \begin{cases} 4 \\ 1 + \frac{3}{4} + \sqrt{3} = \frac{7}{4} + \sqrt{3} \\ 1 + \frac{3}{4} - \sqrt{3} = \frac{7}{4} - \sqrt{3} \end{cases}$$

$$y = 5x - 9 \Rightarrow \underline{y} = \begin{cases} 11 \\ \frac{35}{4} + 5\sqrt{3} - 9 \\ \frac{35}{4} - 5\sqrt{3} - 9 \end{cases}$$

1 YGB - MPI PAPER T - QUESTION 12

$$\underline{x^2} = \begin{cases} 16 \\ \frac{49}{16} + \frac{7\sqrt{3}}{2} + 3 = \frac{97}{16} + \frac{7\sqrt{3}}{2} \\ \frac{49}{16} - \frac{7\sqrt{3}}{2} + 3 = \frac{97}{16} - \frac{7\sqrt{3}}{2} \end{cases}$$

$$\underline{y = \frac{3}{4}x^2 - 4x^{\frac{1}{2}} + 7} = \begin{cases} \frac{3}{4} \times 16 - 4 \times 2 + 7 = 11 \\ \frac{3}{4} \left(\frac{97}{16} + \frac{7\sqrt{3}}{2} \right) - 4 \left(1 + \frac{1}{2}\sqrt{3} \right) + 7 = \frac{483}{64} + \frac{5}{8}\sqrt{3} \\ \frac{3}{4} \left(\frac{97}{16} - \frac{7\sqrt{3}}{2} \right) - 4 \left(1 - \frac{1}{2}\sqrt{3} \right) + 7 = \frac{483}{64} - \frac{5}{8}\sqrt{3} \end{cases}$$

ONLY THE POINT (4,11) LIES ON BOTH CURVE & LINE & THE GRADIENT AT (4,11) IS 5

$\therefore y = 5x - 11$ IS A TANGENT TO THE CURVE AT (4,11)

WE STILL NEED TO SHOW THAT THERE IS NO OTHER INTERSECTION

$$\left. \begin{aligned} y &= 5x - 11 \\ y &= \frac{3}{4}x^2 - 4\sqrt{x} + 7 \end{aligned} \right\} \Rightarrow \begin{aligned} \frac{3}{4}x^2 - 4x^{\frac{1}{2}} + 7 &= 5x - 11 \\ \Rightarrow 3x^2 - 16x^{\frac{1}{2}} + 28 &= 20x - 44 \\ \Rightarrow 3x^2 - 20x - 16x^{\frac{1}{2}} + 64 &= 0 \end{aligned}$$

AS $x=4$ IS A REPEAT ROOT

$$\begin{aligned} \Rightarrow (x-4)^2 \times f(x) &\equiv 3x^2 - 20x - 16x^{\frac{1}{2}} + 64 \\ \Rightarrow (x-4)^2 \times f(x) &\equiv 3[x^2 - 8x + 16] + 4x - 16x^{\frac{1}{2}} + 16 \\ \Rightarrow (x-4)^2 \times f(x) &\equiv 3(x-4)^2 + 4(x - 4x^{\frac{1}{2}} + 4) \\ \Rightarrow (x-4)^2 \times f(x) &\equiv 3(x-4)^2 + 4(\sqrt{x} - 2)^2 \\ \Rightarrow f(x) &\equiv 3 + \frac{4(\sqrt{x} - 2)^2}{(x-4)^2} \end{aligned}$$

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$$\Rightarrow 3x^2 - 20x - 16x^{\frac{1}{2}} + 64 = 0$$

$$\Rightarrow (x-4)^2 \left[3 + \frac{4(\sqrt{x}-2)^2}{(x-4)^2} \right] = 0$$

$$\Rightarrow (x-4)^2 \left(3 + \left(\frac{2(\sqrt{x}-2)}{x-4} \right)^2 \right) = 0$$

\therefore EITHER $x=4$ (REPEATED)

OR ~~$\left(\frac{2(\sqrt{x}-2)}{x-4} \right)^2 = -3$~~

\therefore TANGENT DOES NOT RE-INTERSECT